Math SL Exam Review (Calculator Portion) Name: Key

NC

1) Factorise and solve each quadratic

a) $2x^2 - 30x = 0$ $x=0$ $x=15$
$2x(x-15)=0$

b) $x^2 + 5x - 14 = 0$
$(x+7)(x-2)$
$x=-7, 2$

c) $2x^2 - x - 15 = 0$
$(2x+5)(x-3)$ $x=-5/2$ $x=3$

2) Write $3x^2 + 6x + 10$ in the form $a(x-h)^2 + k$. Give the minimum value.

$3(x+1)^2 + 7$ $y=7$ is the min value

3) Find the value of m for which the quadratic equation $mx^2 - 4x + 2m = 0$ has equal roots

$b^2 - 4ac = 0$
$(-4)^2 - 4(m)(2m) = 0$
$-8m^2 = -16$
$m^2 = 2$
$m = \pm\sqrt{2}$

4) Michele invested 1500 euros at an annual rate of interest of 5.25%, compounded annually.

a) Find the value of Michele's investment after 3 years. Give your answer to the nearest euro.

$A = 1500(1.0525)^3 = \$1749$

b) How many complete years will it take for Michele's initial investment to double in value?

$3000 = 1500(1.0525)^t$
$2 = (1.0525)^t$
$\log 2 = t \log 1.0525$
$t = 13.55$ years

c) What should the interest rate be if Michele's initial investment were to double in value in 10 years?

$2 = (1+r)^{10}$
$1.07177 = 1 + r$
$r = 0.07177$ or 7.2%

5) Solve $\log_{27} x = 1 - \log_{27}(x - 0.4)$

$\log_{27} x + \log_{27}(x - 0.4) = 1$
$x^2 - 0.4x - 27 = 0$
$x = 5.4, -5$
-5 is not possible
$x = 5.4$

NC 6) Solve $5^{4x} = 6e^x$. Write the solution in exact form

$\ln 5^{4x} = \ln 6e^x$
$4x \ln 5 = \ln 6 + \ln e^x$
$4x \ln 5 = \ln 6 + x$
$4x \ln 5 - x = \ln 6$
$x(4\ln 5 - 1) = \ln 6$
$x = \dfrac{\ln 6}{4\ln 5 - 1}$

NC 7) Solve the equation $4^{3x-1} = 64$

$4^{3x-1} = 4^3$ $\quad 3x-1=3$ $\quad x=\frac{4}{3}$

NC 8) Give the largest possible domain for each

a) $f(x) = 8 - x^2 - 10x^3$ b) $g(x) = \frac{1}{2x^2} - \frac{\sqrt{x-3}}{x+1}$ c) $h(x) = \frac{\sqrt[3]{x+6}}{\log_3(x+5)}$

D: All R $\quad$ D: $x \geq 3$ $\quad$ D: $x > -5, x \neq -4$

NC 9) Two functions *f*, *g* are defined as follows:

$f : x \to 3x + 5$

$g : x \to 2(1 - x)$

Find each

a) $f^{-1}(2)$ $\quad -1$

b) $(g \circ f)(-4)$ $\quad 16$

10) For the given geometric sequence, complete the following: $\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} + \cdots$

NC a) Identify the ratio $r = -\frac{2}{3}$

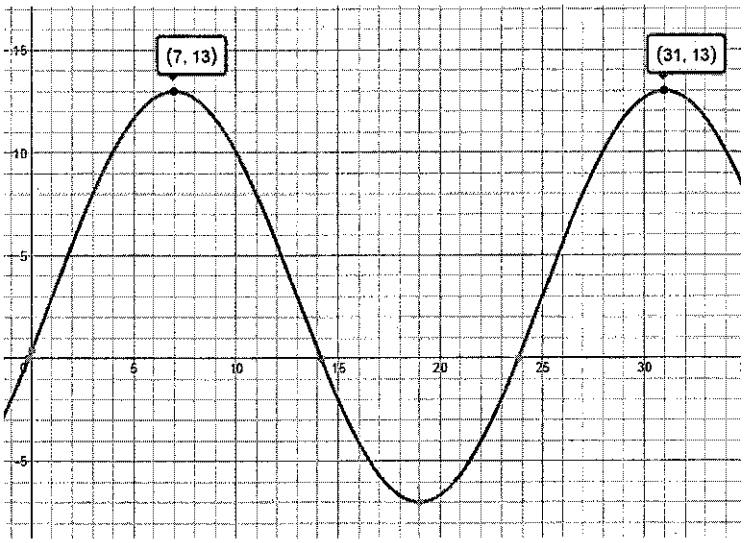
b) Find the 5th term $\frac{32}{243}$

c) Calculate the sum for the infinite series

$S = \frac{2/3}{1-(-2/3)} = \frac{2/3}{5/3} = \frac{2}{5}$

NC

11) Write a sine function for the graph below in the form f(x) = AsinB(x – 1) + C



$f(x) = 10\sin\frac{\pi}{12}(x-1)+3$

12) (a) Factorise the expression $3\sin^2 x - 11\sin x + 6$ $(\sin x - 3)(3\sin x - 2)$

(b) Consider the equation $3\sin^2 x - 11\sin x + 6 = 0$

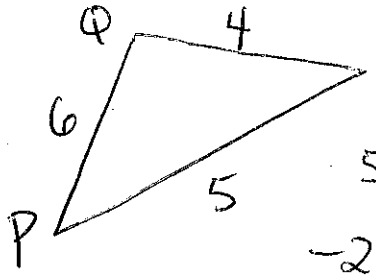
(i) Find the two values of sin x which satisfy the equation $\sin x = 3$, $\sin x = 2/3$

(ii) Solve the equation for $0 \le x \le 180^\circ$

$x = \sin^{-1}(2/3)$

$x = 41.8^\circ, 138.19^\circ$

13) In the triangle PQR, PR = 5 cm, QR = 4 cm and PQ = 6 cm
Calculate



(a) The size of $P\hat{Q}R$;

$P\hat{Q}R = 55.77^\circ$

(b) The area of triangle PQR

$A = \frac{1}{2}(6)(4)\sin 55.77$

$A = 9.92\ cm^2$

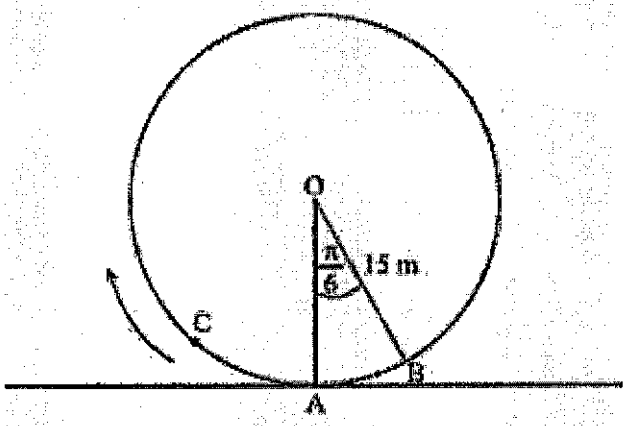
$5^2 = 6^2 + 4^2 - 2(6)(4)\cos Q$

$-27 = -48\cos Q$

$\cos^{-1}(.5625) = 55.77^\circ$

14)

A Ferris wheel with centre O and a radius of 15 metres is represented in the diagram below. Initially seat A is at ground level. The next seat is B, where $A\hat{O}B = \frac{\pi}{6}$.



(a) Find the length of the arc AB. $S = r\theta = \frac{\pi}{6}(15) = \frac{5\pi}{2}$ m

(b) Find the area of the sector AOB. $A = \frac{1}{2}r^2\theta = \frac{1}{2}(15)^2\left(\frac{\pi}{6}\right) = \frac{75\pi}{4}$ m^2

15) Given the sequence 3, 6, 12, 24, ...

(a) Is the sequence arithmetic or geometric?

geometric, constant ratio

(b) Find the value of r $r = 2$

(c) Calculate the 20th term $u_{20} = 3(2)^{20-1} = 1,572,864$

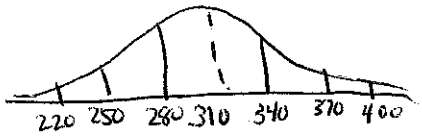
(d) Find the sum of the first 10 terms $S_{10} = \frac{3(1-2^{10})}{1-2} = 3069$

(e) Write a formula for the sum of the first 10 terms in sigma notation

$$\sum_{n=1}^{10} 3(2)^{n-1}$$

16) It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Draw a normal curve and label 3 standard deviations in each direction.



(b) What masses give the middle 95% of all lions?

250 to 370 kg

(c) Calculate the probability that a lion selected a random will have a mass of 350 kg or more.

$Z = \frac{350 - 310}{30} = 1.\overline{3}$

normalcdf(1.3̄, 1E99) = 0.0912

There is a 9.12% chance that a lion will have a mass of 350 kg or more

(d) Calculate the probability that a lion selected a random will have a mass between 295 kg and 400 kg.

$Z = \frac{295 - 310}{30} = -.5$

$Z = \frac{400 - 310}{30} = 3$

normalcdf(-.5, 3) = 0.69

There is a 69% chance that a lion will have a mass between 295 and 400 kg.

(e) How much does a lion have to weigh to be in the top 20% of all lions?

invNorm(.8) = 0.8416

$X = 0.8416(30) + 310 = 335.25$ kg

17) Suppose the probability one of Mr. Stevens' cats has no tail is 0.45. Suppose he has 80 cats in his basement. Calculate each probability

(a) The probability that exactly 10 cats do not have a tail

binompdf(80, 0.45, 10) = $3.75 \times 10^{-10} \approx 0$

(b) The probability that at least 20 cats do not have a tail

1 − binomcdf(80, 0.45, 19) = 0.99993

(c) The probability that fewer than 50 cats DO have a tail

binomcdf(80, .55, 49) = 0.892

18)

The number of hours of sleep of 21 students are shown in the frequency table below.

Hours of sleep	Number of students
4	2
5	5
6	4
7	3
8	4
10	2
12	1

Find each

(a) The median Median: 6

(b) The lower quartile $Q_1 = 5$

(c) The interquartile range $Q_3 - Q_1 = 8 - 5 = 3$

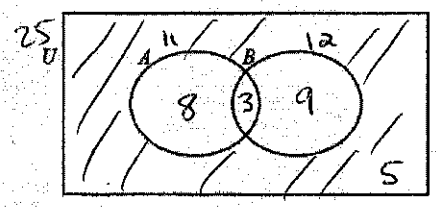
(d) The mean $\bar{x} = 6.76$

(e) The standard deviation $\sigma_x = 2.045$

19)

NC

The following Venn diagram shows a sample space U and events A and B.



$n(U) = 25$, $n(A) = 11$, $n(B) = 12$ and $n(A \cup B)' = 5$.

(a) On the diagram, shade the region $(A \cup B)'$.

(b) Find

(i) $n(A \cap B)$: 3

(ii) $P(A \cap B)$. $\frac{3}{25}$

5. One thousand candidates sit an examination. The distribution of marks is shown in the following grouped frequency table.

Marks	1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80	81–90	91–100
Number of candidates	15	50	100	170	260	220	90	45	30	20

(a) Copy and complete the following table, which presents the above data as a cumulative frequency distribution.

Mark	≤10	≤20	≤30	≤40	≤50	≤60	≤70	≤80	≤90	≤100
Number of candidates	15	65	165	335	595	815	905	950	980	1000

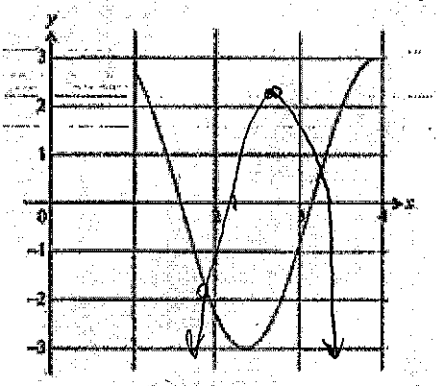
(b) Calculate the mean and standard deviation for the frequency table using the mid-interval.

$\bar{X} = 46.75 \quad \sigma_x = 17.873$

20)

C

Let $f(x) = 3\sin 2x$, for $1 \le x \le 4$ and $g(x) = -5x^2 + 27x - 35$ for $1 \le x \le 4$. The graph of f is shown below.



$X = \frac{-b}{2a} = \frac{-27}{2(-5)} = 2.7$

$Y = -5(2.7)^2 + 27(2.7) - 35 = 1.45$

(a) On the same diagram, sketch the graph of g.

(b) One solution of $f(x) = g(x)$ is 1.89. Write down the other solution. $X = 3.19$

(c) Let $h(x) = g(x) - f(x)$. Given that $h(x) > 0$ for $p < x < q$, write down the value of p and of q.

$(-5x^2 + 27x - 35) - (3\sin 2x)$

$p = 1.89 \quad q = 3.19$

Find the exact values for the following angles:

(a) $\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

(b) $\sin -210° = \frac{1}{2}$

(c) $\cos -\frac{2\pi}{3} = -\frac{1}{2}$

(d) $\tan -630º$ undefined

(e) $\frac{\cos \frac{3\pi}{4}}{\sin\left(-\frac{\pi}{6}\right)} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{1}{2}} = \sqrt{2}$